Analysis of the $[\mathbf{56}, 4^+]$ Baryon Masses in the $1/N_c$ Expansion*

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Abstract

We use the $1/N_c$ expansion of QCD to analyze the spectrum of positive parity resonances with strangeness $S = 0, -1, -2$ and -3 in the 2–3 GeV mass region, supposed to belong to the $[\mathbf{56}, 4^+]$ multiplet. We find that the spin-spin term brings the dominant contribution and that the spin-orbit term is entirely negligible in the hyperfine interaction, in agreement with constituent quark model results. More data are strongly desirable, especially in the strange sector in order to fully exploit the power of this approach.

1 Introduction

Quantum chromodynamics (QCD), the theory of the strong interaction, is an SU(3) gauge theory of quarks and gluons. Unfortunately, because of the complexity of the theory, it is not possible to solve it exactly.

Generally, when a theory cannot be solved directly, one can try to make a perturbative expansion of the theory in terms of its coupling constant. However, the coupling constant g of QCD becomes very large at low energies, too large for a perturbative treatement (see Figure 1).

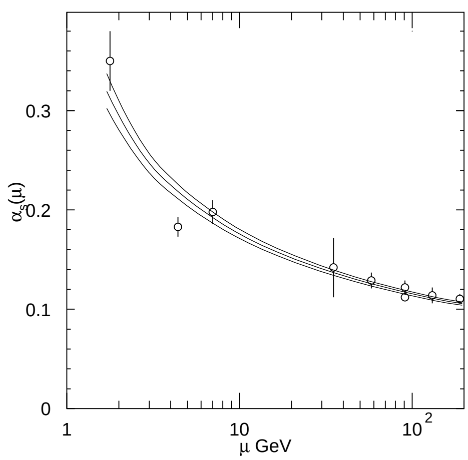

Figure 1 : Values of the strong coupling constant $\alpha_s(\mu) = g^2/4\pi$ vs. the energy scale μ [1].

In 1974, 't Hooft [2] suggested to generalize QCD from three to N_c colors. He found that the inverse of the number of colors could be an expansion parameter of QCD. Later on, based on general arguments, Witten [3] analyzed the properties of mesons and baryon systems in this limit.

The $1/N_c$ expansion of QCD [2, 3] has been proved a useful approach to study baryon spectroscopy. It has been applied to the ground state baryons [4, 5, 6, 7, 8, 9, 10] as well as to excited states, in particular to the negative parity spin-flavor $[\mathbf{70}, 1^-]$-plet ($N = 1$ band) [11, 12, 13, 14, 15, 16, 17], to the positive parity Roper resonance belonging to the $[\mathbf{56'}, 0^+]$-plet ($N = 2$ band) [18] and to the $[\mathbf{56}, 2^+]$-plet ($N = 2$ band) [19]. In this approach the main features of the quark model emerge naturally and in addition new information is provided as for example on the spin-orbit problem.

In this study we explore its applicability to the $[\mathbf{56}, 4^+]$-plet ($N = 4$ band) for the first time. The number of experimentally known resonances in the 2–3 GeV region [1], expected to belong to this multiplet is quite restricted. Among the five possible candidates there are two four-star resonances, $N(2220)9/2^+$ and $\Delta(2420)11/2^+$, one three-star resonance $\Lambda(2350)9/2^+$, one two-star resonance $\Delta(2300)9/2^+$ and one one-star resonance $\Delta(2390)7/2^+$. This is an exploratory study which will allow us to make some predictions.

The aim is to compute the mass operators for the $[\mathbf{56}, 4^+]$ multiplet in a $1/N_c$ expansion.

This paper summarizes the results presented in Refs. [20, 21].

2 Baryons with $N_c = 3$

Baryons are color singlet bound states composed of three quarks[1].

One of the quantum numbers that characterises the

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[1]Here, we do not consider exotic baryons like pentaquarks which are composed of four quarks and one antiquark.

quarks is the flavor. Six different flavors have been observed : u, d, s, c, b, t. Here, we consider only the three first ones.

The total wave function of baryons Ψ is given by

$$\Psi = \psi_{lm}\chi\phi C \tag{1}$$

where ψ_{lm}, χ, ϕ and C are the space, spin, flavor and color parts respectively. The color part C is always antisymmetric, *i.e.* a colorless state. So the remaining part must be symmetric because the total wave function must be antisymmetric as quarks are fermions.

One can introduce the $\mathrm{SU}_f(3)$ symmetry. Physically, if this symmetry is exact the mass of the three different quarks is the same, *i.e.* $m_u = m_d = m_s$.

One can classify baryons with respect to their flavor symmetry properties described by the irreducible representations of the $\mathrm{SU}_f(3)$ group. We obtain the flavor diagrams of Figure 2. Particles belonging to the octet have a spin 1/2. For the decuplet, they have a spin 3/2. With exact $\mathrm{SU}_f(3)$, all the particles in each weight diagram have the same mass.

$n \quad p$

$\Sigma^- \quad \Lambda, \Sigma^0 \quad \Sigma^+$

$\Xi^- \quad \Xi^0$

$\Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++}$

$\Sigma^{*-} \quad \Sigma^{*0} \quad \Sigma^{*+}$

$\Xi^{*-} \quad \Xi^{*0}$

Ω^-

Figure 2: $\mathrm{SU}_f(3)$ flavor diagrams for the octet on the left and for the decuplet on the right.

3 SU(6) Symmetry

One has shown that SU(6) is a good symmetry in $N_c \to \infty$ limit. That means that the first order operator of the expansion of the mass operator will be SU(6) symmetric.

When we consider an exact SU(6) symmetry, we assume that the mass of the particles that belong to the octet is the same that the ones that belong to the decuplet. In fact in SU(6) we have a spin symmetry and a flavor symmetry, *i.e.*

$$SU(6) \supset SU_S(2) \times SU_f(3). \tag{2}$$

The $SU(6)$ operators are :

$$\begin{aligned} S^i &= q^\dagger(S^i \otimes \mathbb{1})q \\ T^a &= q^\dagger(\mathbb{1} \otimes T^a)q \\ G^{ia} &= q^\dagger(S^i \otimes T^a)q \end{aligned} \tag{3}$$

where S^i are the spin generators, T^a the flavor generators, and G^{ia} are the spin-flavor generators.

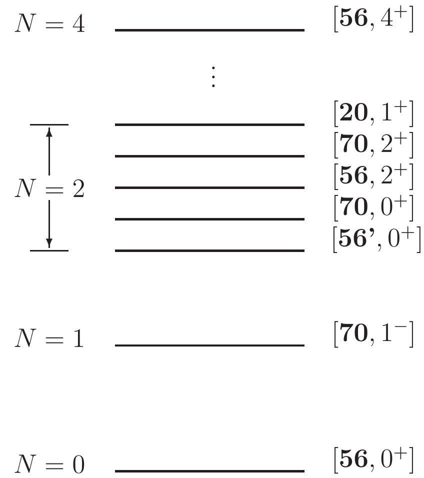

Figure 3: The $N = 0, 1$ and 2 levels of the baryon spectrum.

4 Baryon Spectrum with a Linear Confinement and $N_c = 3$

It is possible to treat the $N_c = 3$ baryon as a system of three non-relativistic quarks bound by a confining potentiel.

In Figure 3, we schematically draw the level sequence of the baryon spectrum up to the $N = 2$ band with a linear confinement. To label the levels, the notation $[\mathbf{x}, \ell^P]$ is used, where $\mathbf{x}$ represents the dimension of the SU(6) irreducible representations, ℓ the angular momentum of the states and P the parity.

5 The Mass Operator

The mass spectrum is calculated in the $1/N_c$ expansion up to and including $\mathcal{O}(1/N_c)$ effects. The mass operator must be rotationally invariant, parity and time reversal even. We consider the $\mathrm{SU}_f(3)$ symmetry breaking to the first order. The isospin breaking is neglected. In term of the mass of the quarks, we assume that $m_u = m_d \neq m_s$.

From the $1/N_c$ expansion, one can write the mass operator as

$$M = \sum_{i=1}^{3} c_i O_i + \sum_{i=1}^{3} b_i \bar{B}_i + \mathcal{O}(1/N_c^2). \tag{4}$$

Here, O_i $(i = 1, 2, 3)$ are $\mathrm{SU}_f(3)$ symmetry operators. The operators $\bar{B}_i$ $(i = 1, 2, 3)$ provide $\mathrm{SU}_f(3)$ breaking and are defined to have vanishing matrix elements for nonstrange baryons. The relation (4) contains the effective coefficients c_i and b_i as parameters. They represent reduced matrix elements that encode the QCD dynamics. The above operators and the values of the corresponding coefficients which we obtained from fitting the experimentally know masses are given in Table 1.

The matrix elements of O_i and $\bar{B}_i$ $(i = 1, 2, 3)$ are given in Tables 2 and 3. The method followed to compute them is presented in Refs. [20, 21].

Operator	Fitted coef. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 = 736 \pm 30$
$O_2 = \frac{1}{N_c} l_i S_i$	$c_2 = 4 \pm 40$
$O_3 = \frac{1}{N_c} S_i S_i$	$c_3 = 135 \pm 90$
$\bar{B}_1 = -\mathcal{S}$	$b_1 = 110 \pm 67$
$\bar{B}_2 = \frac{1}{N_c} l_i G_{i8} - \frac{1}{2\sqrt{3}} O_2$	
$\bar{B}_3 = \frac{1}{N_c} S_i G_{i8} - \frac{1}{2\sqrt{3}} O_3$	

Table 1: Operators of Eq. (4) and coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 0.26$.

	O_1	O_2	O_3
$^2 8_{7/2}$	N_c	$-\frac{5}{2N_c}$	$\frac{3}{4N_c}$
$^2 8_{9/2}$	N_c	$\frac{2}{N_c}$	$\frac{3}{4N_c}$
$^4 10_{5/2}$	N_c	$-\frac{15}{2N_c}$	$\frac{15}{4N_c}$
$^4 10_{7/2}$	N_c	$-\frac{4}{N_c}$	$\frac{15}{4N_c}$
$^4 10_{9/2}$	N_c	$\frac{1}{2N_c}$	$\frac{15}{4N_c}$
$^4 10_{11/2}$	N_c	$\frac{6}{N_c}$	$\frac{15}{4N_c}$

Table 2: Matrix elements of $\mathrm{SU}_f(3)$ singlet operators.

	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$
N_J	0	0	0
Λ_J	1	$\frac{\sqrt{3}\ a_J}{2N_c}$	$-\frac{3\sqrt{3}}{8N_c}$
Σ_J	1	$-\frac{\sqrt{3}\ a_J}{6N_c}$	$\frac{\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{2\sqrt{3}\ a_J}{3N_c}$	$-\frac{\sqrt{3}}{2N_c}$
Δ_J	0	0	0
Σ_J	1	$\frac{\sqrt{3}\ b_J}{2N_c}$	$-\frac{5\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{\sqrt{3}\ b_J}{N_c}$	$-\frac{5\sqrt{3}}{4N_c}$
Ω_J	3	$\frac{3\sqrt{3}\ b_J}{2N_c}$	$-\frac{15\sqrt{3}}{8N_c}$
$\Sigma^8_{7/2} - \Sigma^{10}_{7/2}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Sigma^8_{9/2} - \Sigma^{10}_{9/2}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0
$\Xi^8_{7/2} - \Xi^{10}_{7/2}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Xi^8_{9/2} - \Xi^{10}_{9/2}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0

Table 3: Matrix elements of $\mathrm{SU}_f(3)$ breaking operators. Here, $a_J = 5/2, -2$ for $J = 7/2, 9/2$, respectively and $b_J = 5/2, 4/3, -1/6, -2$ for $J = 5/2, 7/2, 9/2, 11/2$, respectively.

6 Fit and Discussion

The fit of the masses derived from Eq. (4) and the available empirical values used in the fit, together with the corresponding resonance status in the Particle Data Group [1] are listed in Table 4.

The values of the coefficients c_i and b_1 obtained from the fit are presented in Table 1, as already mentioned.

Due to the lack of experimental data in the strange sector it was not possible to include all the operators $\bar{B}_i$ in the fit in order to obtain some reliable predictions. As the breaking of $\mathrm{SU}_f(3)$ is dominated by $\bar{B}_1$ we included only this operator in Eq. (4) and neglected the contribution of the operators $\bar{B}_2$ and $\bar{B}_3$. At a later stage, when more data will hopefully be available, all analytical work performed here could be used to improve the fit. That is why Table 1 contains results for c_i (i = 1, 2 and 3) and b_1 only. The $\chi^2_{\rm dof}$ of the fit is 0.26, where the number of degrees of freedom (dof) is equal to one (five data and four coefficients).

The first column of Table 4 contains the 56 states (each state having a $2I + 1$ multiplicity from assuming an exact SU(2)-isospin symmetry). The columns two to five show the partial contribution of each operator included in the fit, multiplied by the corresponding coefficient c_i or b_1. The column six gives the total mass according to Eq. (4). The errors shown in the predictions result from the errors on the coefficients c_i and b_1 given in Table 1. As there are only five experimental data available, nineteen of these masses are predictions.

The main question is, of course, how reliable is this fit. The answer can be summarized as follows :

- The main part of the mass is provided by the spin-flavor singlet operator O_1, which is $\mathcal{O}(N_c)$.
- The spin-orbit contribution given by $c_2 O_2$ is small. This fact reinforces the practice used in constituent quark models where the spin-orbit contribution is usually neglected.
- The breaking of the SU(6) symmetry keeping the flavor symmetry exact is mainly due to the spin-spin operator O_3. This hyperfine interaction produces a splitting between octet and decuplet states of approximately 130 MeV.
- As it was not possible to include the contribution of $\bar{B}_2$ and $\bar{B}_3$ in our fit, a degeneracy appears between Λ and Σ.

7 Conclusions

In conclusion we have studied the spectrum of highly excited resonances in the 2–3 GeV mass region by describing them as belonging to the $[\mathbf{56}, 4^+]$ multiplet. This is the first study of such excited states based on the $1/N_c$ expansion of QCD. A better description should include multiplet mixing, following the lines developed, for example, in Ref. [22].

Better experimental values for highly excited non-strange baryons as well as more data for the Σ^* and Ξ^* baryons are needed in order to understand the role of the operator $\bar{B}_2$ within a multiplet and for the octet-decuplet mixing. With better data the analytic work performed here will help to make reliable predictions in the large N_c limit formalism.

Acknowledgments

	Partial contribution (MeV)				Total (MeV)	Empirical (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	b_1B_1			
$N_{7/2}$	2209	-3	34	0	2240 ± 97		
$\Lambda_{7/2}$				110	2350 ± 118		
$\Sigma_{7/2}$				110	2350 ± 118		
$\Xi_{7/2}$				220	2460 ± 166		
$N_{9/2}$	2209	2	34	0	2245 ± 95	2245 ± 65	N(2220)****
$\Lambda_{9/2}$				110	2355 ± 116	2355 ± 15	Λ(2350)***
$\Sigma_{9/2}$				110	2355 ± 116		
$\Xi_{9/2}$				220	2465 ± 164		
$\Delta_{5/2}$	2209	-9	168	0	2368 ± 175		
$\Sigma_{5/2}$				110	2478 ± 187		
$\Xi_{5/2}$				220	2588 ± 220		
$\Omega_{5/2}$				330	2698 ± 266		
$\Delta_{7/2}$	2209	-5	168	0	2372 ± 153	2387 ± 88	Δ(2390)*
$\Sigma'_{7/2}$				110	2482 ± 167		
$\Xi'_{7/2}$				220	2592 ± 203		
$\Omega_{7/2}$				330	2702 ± 252		
$\Delta_{9/2}$	2209	1	168	0	2378 ± 144	2318 ± 132	Δ(2300)**
$\Sigma'_{9/2}$				110	2488 ± 159		
$\Xi'_{9/2}$				220	2598 ± 197		
$\Omega_{9/2}$				330	2708 ± 247		
$\Delta_{11/2}$	2209	7	168	0	2385 ± 164	2400 ± 100	Δ(2420)****
$\Sigma_{11/2}$				110	2495 ± 177		
$\Xi_{11/2}$				220	2605 ± 212		
$\Omega_{11/2}$				330	2715 ± 260		

Table 4: Masses (in MeV) predicted by the $1/N_c$ expansion as compared with the empirically known masses. The partial contribution of each operator is indicated for all masses. Those partial contributions in blank are equal to the one above in the same column.

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